

PART 1 — CONSTANTS & KEY VALUES

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$1 \text{ hp} = 746 \text{ W}$$

$$\sqrt{2} \approx 1.414 \quad \sqrt{3} \approx 1.732$$

$$\omega = 2\pi n/60 \text{ [rad/s]} \quad n = 60\omega/2\pi \text{ [rpm]}$$

$$P = \tau \cdot \omega \text{ [W]} \quad \tau = P/\omega \text{ [N}\cdot\text{m]}$$

$$Y: V_L = \sqrt{3} \cdot V_\phi, I_L = I_\phi$$

$$\Delta: V_L = V_\phi, I_L = \sqrt{3} \cdot I_\phi$$

$$P_3\phi = \sqrt{3} \cdot V_L \cdot I_L \cdot \cos\theta = 3 \cdot V_\phi \cdot I_\phi \cdot \cos\theta$$

$$S = \sqrt{3} \cdot V_L \cdot I_L \quad Q = S \cdot \sin\theta \quad PF = \cos\theta = P/S$$

$$\text{Sync speed: } n_s = 120f/P \text{ [rpm]}, \omega_s = 4\pi f/P \text{ [rad/s]}$$

$$50\text{Hz: } 2p \rightarrow 3000, 4p \rightarrow 1500, 6p \rightarrow 1000, 20p \rightarrow 300 \text{ rpm}$$

$$60\text{Hz: } 2p \rightarrow 3600, 4p \rightarrow 1800, 6p \rightarrow 1200 \text{ rpm}$$

$$\text{Slip: } s = (n_s - n_m)/n_s; f_r = s \cdot f_e$$

$$\text{Steel: } \mu_r \approx 2000\text{--}6000, B_{\text{sat}} \approx 1.5\text{--}2\text{T}$$

PART 2 — LECTURE SUMMARIES + CONCEPTUAL Q&A

LECTURE 1 — OVERVIEW OF ELECTRIC MACHINES

An electric machine converts between mechanical and electrical energy. **Motors:** electrical \rightarrow mechanical. **Generators:** mechanical \rightarrow electrical. **Transformers:** change voltage level (no mechanical part). All rely on the same electromagnetic laws (Faraday + Lorentz).

Fleming's rules: Left-hand = motor (creates Force). Right-hand = generator (creates EMF/current). Memory: Left is more "powerful" \rightarrow creates force.

Types of rotating machines: AC machines (synchronous, induction) and DC machines (series, shunt, compound, PMDC). AC machines dominate industry; DC still used for precise speed control.

Q1-TYPE: COMPARE TURBOGENERATOR VS HYDROELECTRIC GENERATOR

	Turbogenerator (Thermal/Nuclear)	Hydro Generator
Speed	High: 3000 rpm (2-pole) or 1500 rpm (4-pole) at 50 Hz	Low: 150–300 rpm (many poles)
Poles	Few (2 or 4)	Many (20, 40, etc.)
Rotor type	Round/non-salient — flush with surface	Salient — sticks out from surface
Shape	Long shaft, small diameter	Short shaft, large diameter
Reason	High speed \rightarrow round rotor avoids mechanical stress	More poles needed \rightarrow larger diameter

Ampere's Law: Electric current creates a magnetic field around it. More turns of wire or more current → stronger H field. This is how electromagnets and motor field windings work.

Faraday's Law: A changing magnetic flux induces an EMF. No change = no EMF. This is the fundamental principle of every electric machine — mechanical motion causes flux change → voltage is generated.

Lenz's Law: Induced current always opposes the change that caused it (conservation of energy). In a motor, this manifests as back-EMF opposing the supply.

Magnetic circuit analogy: Flux flows through a magnetic core like current flows through a wire. mmf (=N·I) is the "voltage source"; reluctance \mathfrak{R} is the "resistance"; flux Φ is the "current". Iron concentrates flux because its permeability is thousands of times higher than air.

Air gap effect: Even a small air gap dominates the total reluctance because $\mu_{r,air}=1$ vs $\mu_{r,iron}=2000+$. This is why the iron core reluctance is often negligible compared to the gap.

Core losses: Hysteresis (energy lost realigning magnetic domains each cycle, $\propto f$) and eddy currents (circulating currents induced by changing B, $\propto f^2$). Laminating the core reduces eddy current paths → less loss.

► Q1-TYPE: WHY ARE CORES LAMINATED? WHAT ARE THE TWO TYPES OF CORE LOSSES?

Cores are laminated to break up the large eddy current loops that form in solid iron when subjected to alternating flux. Each thin lamination is insulated from neighbours, forcing eddy currents into much smaller loops → greatly reduced eddy current losses ($P_E \propto f^2 B^2$). The two core losses are: (1) **Hysteresis loss** ($P_H \propto f \cdot B^n$) — energy to repeatedly align magnetic domains; (2) **Eddy current loss** ($P_E \propto f^2 \cdot B^2$) — circulating currents in the conductive core.

Ampere's Law: $\oint H \cdot dl = \sum i \rightarrow H \cdot l_c = N \cdot i$ (uniform core)

Faraday's Law: $e = -N \cdot d\Phi/dt$ $e = BLv$ (moving bar)

B-H: $B = \mu_0 \mu_r \cdot H$ $\Phi = B \cdot A$ [Wb]

mmf: $F = N \cdot i$ [A·turns]

Reluctance: $\mathfrak{R} = l/(\mu_0 \mu_r A)$ [A·t/Wb]

Air gap: $\mathfrak{R}_g = g/(\mu_0 \cdot A_g)$

Ohm's law (magnetic): $\Phi = \text{mmf}/\mathfrak{R}_{\text{total}}$

Series: $\mathfrak{R}_{\text{total}} = \mathfrak{R}_{\text{core}} + \mathfrak{R}_{\text{gap}}$

Flux linkage: $\lambda = N \cdot \Phi$ $L = N^2/\mathfrak{R}$ $e = L \cdot di/dt$

Toroid: $B = \mu_0 \mu_r NI/(2\pi r)$

Hysteresis: $P_H = K_H \cdot f \cdot B_{\text{max}}^n$

Eddy current: $P_E = K_E \cdot f^2 \cdot B_{\text{max}}^2$

Magnetic ↔ Electric Analogy

Electric	Magnetic
EMF (V)	mmf = Ni
Current (I)	Flux (Φ)
Resistance (R)	Reluctance (ℜ)
KCL: ΣI=0 at node	ΣΦ=0 at node
KVL: ΣV=0 in loop	Σ(Ni)=ΣΦℜ in loop

Structure: Stator = field (creates static B field, via PM or field winding). Rotor = armature (carries current, rotates). Commutator + brushes reverse current direction as rotor turns so the torque always acts in the same direction.

Back-EMF self-regulation: As the motor speeds up, $E_a = K_e \Phi \omega$ increases, reducing the armature current $I_a = (V_a - E_a)/R_a$. This automatically balances torque to the load — no external controller needed for basic operation.

Types and their behaviour:

- **Separately excited / Shunt:** Field and armature are independent (or in parallel). Speed is nearly constant — slight droop with load. Good for precise speed control.
- **Series:** Field winding in series with armature $\rightarrow \Phi \propto I_a \rightarrow \tau \propto I_a^2$. Huge starting torque. Speed inversely relates to torque (hyperbolic). Speed $\rightarrow \infty$ if load removed. **Never run unloaded.**
- **Cumulative compound:** Series + shunt fields in same direction. Best of both — high starting torque and safe at no-load.
- **Differential compound:** Series field opposes shunt field. Unstable, never used.
- **PMDC:** Field from permanent magnets. Efficient, compact, no field loss. Cannot control flux.

Speed control: Armature voltage control (below base speed, constant torque). Field weakening / flux control (above base speed, constant power, less torque). Combined gives 40:1 speed range. Armature resistance control is simple but wastes power.

DC Motor Starter: At standstill, $E_a = 0 \rightarrow I_{start} = V/R_a$ can be 20× rated current. Must insert series resistance at start and remove as motor accelerates.

► Q1-TYPE: TWO METHODS OF DC MOTOR SPEED CONTROL AND THEIR LIMITATIONS

Armature voltage control (below base speed): Reduce $V_a \rightarrow$ lower ω . Shape of torque-speed unchanged. Maximum torque constant. Cannot go above base speed (would need dangerously high voltage).

Field flux control (above base speed): Reduce $I_f \rightarrow$ lower $\Phi \rightarrow$ higher ω . No-load speed increases; curve slope flattens. Torque capability decreases. Cannot go below base speed (would need too high field current).

Combined: armature control below base + field control above base \rightarrow very wide speed range.

► Q1-TYPE: WHY NEVER RUN A SERIES DC MOTOR UNLOADED?

In a series motor, $\Phi \propto I_a \propto$ load current. As load $\rightarrow 0$, $I_a \rightarrow 0$, $\Phi \rightarrow 0$. From $\omega = E_a/(K_e \Phi)$, as $\Phi \rightarrow 0$ the speed $\rightarrow \infty$. The motor over-speeds and can mechanically destroy itself. Never operate series motors unloaded or with belt-drive that might slip.

Back-EMF: $E_a = K_e \cdot \Phi \cdot \omega_m$ [V]

Torque: $\tau = K_e \cdot \Phi \cdot I_a$ [N·m] (same K_e !)

Motor KVL: $V_a = E_a + I_a \cdot R_a$ (+ $I_a R_f$ for series)

Generator KVL: $V_a = E_a - I_a \cdot R_a$

Developed power: $P_d = E_a \cdot I_a = \tau \cdot \omega_m$

Shunt: $I_L = I_a + I_f$; $V_T = V_a$

Series: $I_L = I_a = I_f$

T- ω (shunt): $\omega = V_a/(K_e \Phi) - R_a/(K_e \Phi)^2 \cdot \tau$

Series E ratio: $E_2/E_1 = (I_{a2} \cdot \omega_2)/(I_{a1} \cdot \omega_1)$

Losses: $P_a = I_a^2 R_a$, $P_f = I_f^2 R_f$

Brush: $P_{BD} = V_{BD} \cdot I_a$ ($V_{BD} \approx 2V$)

P_{rot} from no-load test

$P_{out} = P_d - P_{rot} - P_{core}$

$\eta = P_{out}/P_{in}$; $P_{in} = V_T \cdot I_L$

Good PM material: Large B_{res} (strong flux) + large H_c (hard to demagnetize). BH_{max} = area of largest rectangle inscribed in B-H curve.

Best: Nd-Fe-B, Sm-Co (rare-earth).

LECTURE 4 — AC MACHINE FUNDAMENTALS

Rotating magnetic field: Three-phase currents (equal magnitude, 120° apart in time) flowing in three windings (120° apart in space) produce a magnetic field that rotates at constant speed and constant magnitude. This is the engine of all AC machines.

Why it works: At any instant, the three field contributions (each oscillating) add as vectors to a resultant that always has magnitude $1.5B_M$ and rotates at the supply electrical frequency. Swapping any two phases reverses the direction of rotation.

Synchronous speed: The field rotation speed depends on supply frequency and pole count ($n_s = 120f/P$). More poles \rightarrow slower rotation for the same frequency. This is why slow hydro generators need many poles.

Induced voltage: The rotating field sweeps past stator conductors \rightarrow sinusoidal EMF. Magnitude proportional to flux, speed, and number of turns. Three windings produce three voltages 120° apart — a balanced 3-phase set.

Harmonics problem: Real flux density distributions aren't perfectly sinusoidal \rightarrow voltage harmonics. Fractional-pitch windings (pitch factor k_p) and distributed windings (distribution factor k_d) reduce harmonics. Winding factor $k_w = k_p \cdot k_d$.

Q1-TYPE: WHAT IS THE RELATIONSHIP BETWEEN ELECTRICAL FREQUENCY AND MECHANICAL SPEED?

$n_s = 120f/P$, where P = number of poles. For a 2-pole machine, one electrical cycle = one mechanical revolution. For a 4-pole machine, one electrical cycle = half a mechanical revolution. In general: $f_e = (P/2) \times f_{\text{mech}} = (P/2) \times n/60$. This means at 50 Hz, a 2-pole machine runs at 3000 rpm, a 4-pole at 1500 rpm, etc.

3-phase currents: $i_a = I_M \sin(\omega t)$, $i_b = I_M \sin(\omega t - 120^\circ)$, $i_c = I_M \sin(\omega t - 240^\circ)$

Net B field: $|B_{\text{net}}| = 1.5 \cdot B_M$ (constant), rotates at ω_e

Sync speed: $n_s = 120f/P$ [rpm]

Elec-mech angle: $\theta_e = (P/2) \cdot \theta_m$

Peak induced voltage: $\hat{e} = N_C \cdot \omega_m \cdot \Phi_M$ [V]

RMS phase voltage: $V_\phi = \hat{e}/\sqrt{2}$

Y terminal voltage: $V_T = \sqrt{3} \cdot V_\phi$

Winding factor: $k_w = k_p \cdot k_d \leq 1$

Reverse rotation: Swap any two of the three supply phases. The rotating B field direction reverses \rightarrow motor spins opposite way.

LECTURE 5 — SYNCHRONOUS MACHINES

Synchronous generator operation: DC current in rotor winding creates a rotor magnetic field. Prime mover spins rotor at synchronous speed \rightarrow rotating B field \rightarrow induces 3-phase AC voltages in stator. Rotor MUST spin at exactly $n_s = 120f/P$ to produce rated frequency.

Armature reaction: When a load is connected, stator currents create their own magnetic field, distorting the air-gap field. For lagging loads (inductive), this demagnetizes the machine $\rightarrow V_T$ drops. For leading loads (capacitive), it magnetizes $\rightarrow V_T$ rises. Modelled by synchronous reactance X_S .

Equivalent circuit: E_A (internal voltage, proportional to field current) in series with X_S (and small R_A). KVL gives $E_A = V_\phi + I_A \cdot jX_S$ (generator). This model captures all steady-state behaviour.

When steam is cut off (generator \rightarrow motor): Without prime mover torque, rotor slows and falls behind the rotating stator field. Torque angle δ reverses sign. Machine now draws power from the grid to maintain synchronous speed — it becomes a synchronous motor automatically.

Synchronous motor V-curves: At fixed load, varying field current changes the reactive power exchange. Underexcited (low I_F) = lagging PF = absorbs Q. Unity PF at minimum I_A . Overexcited (high I_F) = leading PF = supplies Q to grid. Overexcited synchronous motors are used as "synchronous condensers" for power factor correction.

Motor cannot start direct-on-line: At standstill, rotor field and stator field are at different speeds \rightarrow alternating torque averages to zero. Must use: variable frequency drive (reduce frequency from zero), external motor to bring to sync speed, or amortisseur (damper) windings (acts as induction motor to pull near sync speed, then "snaps in").

► Q1-TYPE: WHAT IS ARMATURE REACTION?

Armature reaction is the distortion of the air-gap magnetic field caused by the current flowing in the stator (armature) windings. At no-load, the field is purely from the rotor. When a load is connected, stator currents create their own field that adds to or subtracts from the rotor field depending on power factor. For lagging PF loads: stator field opposes rotor field → net flux reduces → V_T drops. For leading PF: stator field aids rotor field → V_T rises.

► Q1-TYPE: COMPARE CONCENTRATED VS DISTRIBUTED STATOR WINDINGS

	Concentrated	Distributed
Back-EMF	Trapezoidal	Sinusoidal
Torque ripple	Higher	Lower
End-turns	Shorter (less copper)	Longer (more copper)
Harmonics	More	Less
Application	BLDC, servo motors	High-speed, high-efficiency sync motors

► Q1-TYPE: WHAT HAPPENS WHEN STEAM IS CUT OFF FROM A SYNC GENERATOR?

Without prime mover torque, the rotor decelerates and the torque angle δ swings from positive (generating) to negative. The machine now draws electrical power from the grid to maintain synchronous speed, converting it to mechanical power to overcome losses — it acts as a synchronous motor. If lossless, it draws only magnetizing current (reactive power) at unity or lagging PF with no real power.

Internal voltage: $E_A = K \cdot \Phi \cdot \omega$ [V rms]

Generator KVL: $E_A = V_\phi + I_A \cdot (R_A + jX_S)$

Motor KVL: $V_\phi = E_A + I_A \cdot (R_A + jX_S)$

→ $E_A = V_\phi - I_A \cdot jX_S$ (ignore R_A)

Power: $P = 3 \cdot V_\phi \cdot E_A \cdot \sin\delta / X_S$

P_{\max} ($\delta=90^\circ$): $P_{\max} = 3 \cdot V_\phi \cdot E_A / X_S$

Torque: $\tau = P_{\text{conv}} / \omega_m$

τ_{\max} (motor): $= 3 \cdot V_\phi \cdot E_A / (X_S \cdot \omega_m)$

DC test (Y): $R_A = V_{DC} / (2 \cdot I_{DC})$

OC test (Y): $E_A = V_{OC, \text{line}} / \sqrt{3}$

SC test: $I_{A, SC} = I_{SC, \text{line}} \text{ (Y)}$

$Z_S = E_A / I_{A, SC}$; $X_S = \sqrt{Z_S^2 - R_A^2}$

Phasor rules (generator, V_ϕ =reference $\angle 0^\circ$):

Lagging load: $I_A \angle -\theta \rightarrow E_A = V_\phi + I_A \cdot jX_S \rightarrow |E_A| > V_\phi$ (V_T drops)

Leading load: $I_A \angle +\theta \rightarrow |E_A| < V_\phi$ (V_T rises)

Grid sync — all 4 must hold:

1. Same RMS voltage
2. Same phase sequence
3. Same phase angle
4. Generator f slightly higher than grid

4 Operating Modes:

Mode	Real P	Reactive Q
Gen + overexcited	Output	Supply to grid
Gen + underexcited	Output	Absorb from grid
Motor + overexcited	Input	Supply to grid (leading PF)
Motor + underexcited	Input	Absorb from grid (lagging PF)

Generator: E_A leads V_ϕ . Motor: E_A lags V_ϕ .

How it works: Stator creates rotating magnetic field (same as L4). This field cuts stationary rotor conductors → induces voltages → induces currents in shorted rotor → rotor current in field creates a force (torque) that drags the rotor after the field.

Why slip is necessary: If the rotor ran at exactly synchronous speed, there would be no relative motion between rotor and stator field → no induced voltage → no rotor current → no torque. The rotor would slow down. In normal operation, slip = 1–5%: small, but essential.

Like a transformer: Stator = primary; rotor = secondary. Stator induces voltage into rotor. Unlike a transformer, secondary frequency depends on slip: $f_r = s \cdot f_e$. At standstill $s=1$ (full frequency); at rated speed $s \approx 0.03$ (very low frequency). Rotor reactance also depends on slip: $X_R = s \cdot X_{R0}$.

Torque-speed curve: Zero torque at synchronous speed ($s=0$). Increases as rotor slows. Peaks at pullout/breakdown torque ($s=s_{\max}$). Then decreases to starting torque ($s=1$). Stable operating region is between $s=0$ and $s=s_{\max}$ (nearly linear).

Key insight about R_2 : Increasing rotor resistance R_2 shifts the peak torque to higher slip (lower speed), but the peak torque value stays the same. This can be exploited in wound-rotor motors to get maximum starting torque at $s=1$ by setting $R_2 = \sqrt{(R_{TH})^2 + (X_{TH} + X_2)^2}$.

Power flow chain: P_{in} → subtract stator Cu loss and core loss → P_{AG} (crosses air gap) → subtract rotor Cu loss ($= s \cdot P_{AG}$) → $P_{conv} = (1-s) \cdot P_{AG}$ → subtract friction/windage → P_{out} .

Speed control: Most modern: VVVF (Variable Voltage Variable Frequency) inverter. Keep V/f constant below base speed (constant torque). Above base speed: keep V constant, increase f (constant power, field weakening).

Q1-TYPE: WHY CAN AN INDUCTION MOTOR NEVER REACH SYNCHRONOUS SPEED?

Torque is produced only because the stator field rotates faster than the rotor — this relative motion induces rotor currents. If the rotor reached synchronous speed, relative motion = 0 → no induced EMF → no rotor current → no torque. Friction losses would then slow the rotor below synchronous speed again. Induction motors always run below n_s , typically 1–5% below at full load.

Q1-TYPE: CAGE ROTOR VS WOUND ROTOR — DIFFERENCES

Cage rotor: Conducting bars shorted at ends by rings. No slip rings. Cheap, robust, low maintenance. Cannot access rotor circuit externally. Most widely used type.

Wound rotor: Full 3-phase winding, connected to slip rings. External resistance can be inserted → smooth starting with high torque. More expensive, needs brush maintenance. Mostly replaced by cage + VFD.

Q1-TYPE: EFFECT OF INCREASING ROTOR RESISTANCE R_2

- s_{\max} (slip at peak torque) increases proportionally to R_2
- τ_{\max} (peak torque value) is unchanged
- Starting torque τ_{start} increases (up to maximum when $s_{\max}=1$)
- Motor runs at higher slip for same torque → less efficient

Slip: $s = (n_s - n_m) / n_s$; $n_m = (1-s)n_s$

Rotor freq: $f_r = s \cdot f_e$

Rotor voltage: $E_R = s \cdot E_{R0}$; $X_R = s \cdot X_{R0}$

Rotor current (referred): $I_2 = E_1 / (R_2/s + jX_2)$

Power flow:

$$P_{in} = 3V_1 I_1 \cos\theta$$

$$P_{SCL} = 3I_1^2 R_1 \text{ (stator Cu loss)}$$

$$P_{AG} = P_{in} - P_{SCL} - P_{core} = 3I_2^2 R_2 / s = \tau_{ind} \cdot \omega_s$$

$$P_{RCL} = s \cdot P_{AG} = 3I_2^2 R_2 \text{ (rotor Cu loss)}$$

$$P_{conv} = (1-s) \cdot P_{AG} = \tau_{ind} \cdot \omega_m$$

$$P_{out} = P_{conv} - P_{rot}; \tau_{load} = P_{out} / \omega_m$$

Thevenin equivalent:

$$V_{TH} \approx V_1 \cdot X_M / (X_1 + X_M)$$

$$R_{TH} \approx R_1 \cdot (X_M / (X_1 + X_M))^2; X_{TH} \approx X_1$$

$$\tau_{ind} = 3V_{TH}^2 \cdot (R_2/s) / [\omega_s \cdot \{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2\}]$$

$$s_{max} = R_2 / \sqrt{(R_{TH}^2 + (X_{TH} + X_2)^2)}$$

$$\tau_{max} = 3V_{TH}^2 / [2\omega_s (R_{TH} + \sqrt{(R_{TH}^2 + (X_{TH} + X_2)^2})])]$$

$$\tau_{max} \text{ INDEPENDENT of } R_2; s_{max} \text{ PROPORTIONAL to } R_2$$

$$\text{DC test: } R_1 = V_{DC} / (2 \cdot I_{DC}) \text{ [Y-conn]}$$

$$\text{No-load test: gives } P_{rot} + P_{core} \text{ and } X_M$$

$$\text{Locked-rotor: } R_{LR} = P_{LR} / (3I^2); Z_{LR} = V_T / (\sqrt{3} \cdot I)$$

$$X_{LR} = \sqrt{(Z_{LR}^2 - R_{LR}^2)} \cdot (f_{rated} / f_{test})$$

$$R_2 = R_{LR} - R_1; \text{ Class A/D: } X_1 = X_2 = X_{LR} / 2$$

$$\text{Key ratios: } P_{RCL} / P_{AG} = s \mid P_{conv} / P_{AG} = 1 - s \mid \tau_{ind} = P_{AG} / \omega_s$$

LECTURE 7 — SPECIAL MACHINES & APPLICATIONS

Universal motor: A series DC motor operating on AC supply. Because field and armature currents both reverse simultaneously each half-cycle, the torque ($\propto I_a^2$) remains always positive. Completely laminated to reduce core losses at AC frequency. Runs slower on AC than DC at same current (reactance drops voltage, reducing E_A). Speed controlled by TRIAC/SCR varying V_{rms} . Applications: vacuum cleaners, drills, food mixers (high torque, compact, intermittent).

BLDC (Brushless DC): PM DC motor "turned inside-out" — high-power winding on stator; PMs on rotor. Electronic inverter + Hall sensors replace mechanical commutator. Each phase conducts 120° per half-cycle; only two phases conduct at any instant → constant power = 2EI. Standard DC equations apply ($E_a = k\omega$, $\tau = kI$). Advantages over conventional DC: no brush wear, higher efficiency, higher possible speeds, less RF noise.

Switched Reluctance Machine (SRM): Doubly-salient structure (salient poles on both stator AND rotor). No PMs, no rotor windings — extremely robust, suitable for high-speed and harsh environments. Torque produced by tendency of rotor to minimize reluctance path ($\tau = \frac{1}{2} I^2 \cdot dL/d\theta$). Needs electronic commutation (must switch off before full alignment to avoid negative torque). Drawbacks: high torque ripple, acoustic noise.

Single-phase induction motor starting: A pure single-phase supply creates two equal counter-rotating fields → zero net starting torque. Must create a phase shift to simulate a rotating field. Methods: capacitor-start (cheapest practical method, auxiliary winding + series capacitor ≈ 90° shift), capacitor-start capacitor-run (two capacitors, better running PF), split-phase (different R/X auxiliary winding), shaded-pole (copper ring over part of each pole, cheapest, lowest efficiency).

Q1-TYPE: COMPARE BLDC AND SYNCHRONOUS AC MOTORS

Feature	BLDC	Sync AC Motor
Back-EMF shape	Trapezoidal	Sinusoidal
Drive current shape	Quasi-square (DC-like)	Sinusoidal (AC)
Torque ripple	Higher	Lower
Noise	More audible/electrical	Quieter
Stator winding	Concentrated (trapezoidal)	Distributed (sinusoidal)
Similarities	Both brushless, PM rotor, 3-phase stator	

Q1-TYPE: WHY IS BLDC BETTER THAN CONVENTIONAL PM DC MOTOR?

- No brushes/commutator → no sparking, no maintenance, no RFI
- Higher efficiency (eliminates brush friction and contact voltage drop)
- Longer operational life and higher reliability
- Higher speeds possible (>50,000 rpm)
- Better heat dissipation (power winding on stator, easily cooled)

Universal motor:

$$Z = (R_a + R_f) + j \cdot 2\pi f \cdot (L_a + L_f)$$

$$|V_s|^2 = (E_a + I_a R_{total})^2 + (I_a X_{total})^2$$

$$\tau \propto I_a^2 \rightarrow \text{half torque: } I_{a,new} = I_{old} \cdot \sqrt{0.5}$$

$$\Phi \propto I_a \text{ (series), so } E_a \propto I_a \cdot \omega$$

BLDC (2 phases active at once):

$$V_s = E_a + 2 \cdot I \cdot R_{ph} (+2V_{os} \text{ switch drop})$$

$$P_{conv} = 2 \cdot E \cdot I \text{ (constant, ripple-free)}$$

$$E_a = k \cdot \omega \quad \tau = k \cdot I \text{ (same as DC motor)}$$

SRM torque: $\tau = \frac{1}{2} \cdot I^2 \cdot dL/d\theta$

Switch current off BEFORE rotor aligns (else negative torque!)

Single-phase IM starting:

- Capacitor-start: aux winding + capacitor $\approx 90^\circ$ shift; disconnects at speed
- Capacitor-start cap-run: 2 caps, better running PF
- Split-phase: aux winding with higher R/X; simpler
- Shaded-pole: Cu ring over pole; cheapest, least efficient

PART 3 — SOLVING STEPS**A — Magnetic Circuit**

1. Convert all dims to SI. Draw \mathfrak{R} analogy (series/parallel).
2. $\mathfrak{R}_{core} = l_c / (\mu_0 \mu_r A_c)$; $\mathfrak{R}_{gap} = g / (\mu_0 A_g)$; $\mathfrak{R}_{total} = \mathfrak{R}_{core} + \mathfrak{R}_{gap}$.
3. $\Phi = N \cdot I / \mathfrak{R}_{total}$. $B = \Phi / A$. $\lambda = N \cdot \Phi$. $L = \lambda / I = N^2 / \mathfrak{R}$.
4. Verify: $H_c \cdot l_c + H_g \cdot g = N \cdot I$ ($H = B / \mu_0 \mu_r$).

B — DC Motor Calculation

1. Identify type. Series: $I_a = I_f = I_L$. Shunt: $I_L = I_a + I_f$, $V_a = V_T$.
2. $E_a = V_a - I_a (R_a + R_f)$ [motor]; $E_a = V_a + I_a R_a$ [gen].
3. $P_d = E_a \cdot I_a$. $\omega = 2\pi n / 60$. $\tau = P_d / \omega$.
4. Losses $\rightarrow P_{out} = P_d - P_{rot}$. $\eta = P_{out} / (V_T \cdot I_L)$.
5. Series speed ratio: $E_2 / E_1 = (I_{a2} \cdot \omega_2) / (I_{a1} \cdot \omega_1)$.

C — AC Fundamentals

1. $n_s = 120f / P$; $\omega_m = 2\pi n_s / 60$.
2. $\Phi = 2 \cdot r \cdot l \cdot B_M$. $\hat{e} = N_C \cdot \omega_m \cdot \Phi_M$.
3. $V_{\phi,rms} = \hat{e} / \sqrt{2}$. Y: $V_T = \sqrt{3} \cdot V_{\phi}$.

D — Sync Generator

1. Y: $V_{\phi} = V_T / \sqrt{3}$. Δ : $V_{\phi} = V_T$.
2. $I_A = S / (3V_{\phi})$ or $P / (3V_{\phi} \cos\theta)$. Lagging $\rightarrow I_A \angle -\theta$; leading $\rightarrow I_A \angle +\theta$.
3. $E_A = V_{\phi} + I_A \cdot jX_S$ (complex). $V_{\phi} = V_{\phi} \angle 0^\circ$ as reference.
4. $|E_A|$, δ , $P = 3V_{\phi} E_A \sin\delta / X_S$.
5. OC/SC: $R_A = V_{DC} / (2I_{DC})$; $E_A = V_{OC} / \sqrt{3}$; $X_S = E_A / I_{SC}$.

E — Sync Motor (Load Change)

1. $P_{in} = P_{out}(\text{hp} \times 746) + \text{losses}$. $I_A = P_{in} / (3V_\phi \cos\theta)$. Lagging $\rightarrow -\theta$.
2. $E_A = V_\phi - I_A \cdot jX_S \rightarrow$ find $|E_A|$ and δ .
3. Load change: $|E_A| = \text{const}$. $P_2/P_1 = \sin\delta_2/\sin\delta_1 \rightarrow$ find $\delta_2 \rightarrow$ new I_A from KVL.
4. $\text{PF} = \cos(\angle I_A \text{ vs } V_\phi)$. Leading if I_A has +ve angle.

F — Sync Motor (Field Change)

1. $P = \text{const} \rightarrow E_A \sin\delta = \text{const}$.
2. New $|E_A| = k \cdot E_{A, \text{old}}$ (e.g. +25% $\rightarrow \times 1.25$).
3. $E_{A, \text{new}} \sin\delta_{\text{new}} = E_{A, \text{old}} \sin\delta_{\text{old}} \rightarrow$ find δ_{new} .
4. New $I_A = (V_\phi - E_{A, \text{new}} \angle -\delta_{\text{new}}) / (jX_S)$. Find new PF.

G — Induction Motor Full Calculation

1. $n_s = 120f/P$; $\omega_s = 2\pi n_s/60$; $n_m = (1-s)n_s$; $\omega_m = (1-s)\omega_s$.
2. $V_{TH} \approx V_1 \cdot X_M / (X_1 + X_M)$. $Z_{rotor} = R_2/s + jX_2$. $Z_F = jX_M \parallel Z_{rotor}$. $Z_{total} = (R_1 + jX_1) + Z_F$.
3. $I_1 = V_1 / Z_{total}$. $\text{PF} = \cos(\angle Z_{total})$. $P_{in} = 3V_1 I_1 \cos\theta$.
4. $I_2 = V_{TH} / \sqrt{(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2}$. $P_{AG} = 3I_2^2 R_2/s$.
5. $\tau_{ind} = P_{AG} / \omega_s$. $P_{conv} = (1-s)P_{AG}$. $P_{out} = P_{conv} - P_{rot}$. $\tau_{load} = P_{out} / \omega_m$.
6. $\eta = P_{out} / P_{in} \times 100\%$.

H — Induction Motor Max Torque

1. $s_{max} = R_2 / \sqrt{(R_{TH})^2 + (X_{TH} + X_2)^2}$. Speed $= n_s(1-s_{max})$.
2. If R_2 doubled $\rightarrow s_{max}$ doubled; τ_{max} unchanged; starting torque changes.
3. Starting torque: substitute $s=1$ into τ_{ind} formula.

I — Induction Motor Parameter Tests

1. DC test: $R_1 = V_{DC} / (2I_{DC})$.
2. No-load: $P_{rot} + P_{core} = P_{NL} - 3I_{NL}^2 R_1$. $X_1 + X_M = V_1 / I_{NL}$.
3. Locked-rotor: $R_{LR} = P_{LR} / (3I^2)$; $Z_{LR} = V_T / (\sqrt{3} \cdot I)$; $X_{LR} = \sqrt{(Z^2 - R^2) \cdot (f_{rated} / f_{test})}$. $R_2 = R_{LR} - R_1$. Split X equally (Class A/D).
4. Find s_{max} and τ_{max} using computed parameters.

J — Universal Motor

1. $Z = (R_a + R_p) + j2\pi f(L_a + L_p)$. At full load: find E_a from $|V_s|^2 = (E_a + I_a R)^2 + (I_a X)^2$.
2. $K_e \Phi = E_a / \omega$ (since $\Phi \propto I_a$: $K_e \cdot K_f = E_a / (I_a \cdot \omega)$).
3. Half torque: $\tau \propto I_a^2 \rightarrow$ new $I_a = I_{a, \text{old}} \cdot \sqrt{(\tau_{new} / \tau_{old})}$.
4. New E_a from voltage equation. New $\omega = E_{a, \text{new}} / (K_e K_f I_{a, \text{new}})$.

PART 4 — EXAM TRAPS & QUICK REFERENCE

Do NOT Get These Wrong

- **P = poles (NOT pole-pairs).** $n_s = 120f/P$
- DC test Y-conn: $R_A = V_{DC}/2I_{DC}$ (2 windings in series)
- Δ -conn: $I_A = I_{\text{phase}}$; $I_{\text{line}} = \sqrt{3} \cdot I_{\text{phase}}$
- ω must be rad/s in $P = \tau \cdot \omega$
- 1 hp = 746 W exactly
- Induction motor: CANNOT reach n_s
- Sync motor: CANNOT start direct-on-line
- Series DC: NEVER run unloaded
- τ_{max} of IM: independent of R_2
- Generator KVL: $E_A = V_\phi + I_A Z$; Motor: $V_\phi = E_A + I_A Z$
- $V_\phi = V_T / \sqrt{3}$ only for Y-connection

Key Relationships

$$E_a(\text{DC}) = K_e \Phi \omega; \tau(\text{DC}) = K_e \Phi I_a \text{ (same } K_e \text{)}$$

$$P_{AG} = \tau_{\text{ind}} \cdot \omega_s \text{ (stator-side perspective)}$$

$$P_{\text{conv}} = \tau_{\text{ind}} \cdot \omega_m \text{ (rotor-side perspective)}$$

$$P_{RCL} = s \cdot P_{AG}; P_{\text{conv}} = (1-s)P_{AG}$$

$$P_{RCL}/P_{\text{conv}} = s/(1-s)$$

$$\tau \propto V_1^2 \text{ (induction motor)}$$

$$s_{\text{max}} \propto R_2; \tau_{\text{max}} \text{ independent of } R_2$$

$$\text{Sync gen: } P = 3V_\phi E_A \sin \delta / X_S$$

$$\text{Sync motor: } P_{\text{max}} = 3V_\phi E_A / X_S \text{ (at } \delta = 90^\circ \text{)}$$

Machine Quick Compare

	DC	Sync	Induction
Speed	Variable	Fixed= n_s	Below n_s
Start DOL	w/ starter	No	Yes
PF control	N/A	Yes (I_F)	No (always lag)
Brushes	Yes	Slip rings	No (cage)
Q behavior	—	Supply or absorb	Always absorbs
Maint.	High	Med	Low

Induction Motor Power Flow Chain (Memorise)

$$P_{\text{in}} = 3 \cdot V_1 \cdot I_1 \cdot \cos \theta$$

$$- P_{SCL} = 3I_1^2 R_1 \text{ (stator copper)}$$

$$- P_{\text{core}} \text{ (core losses)}$$

$$= P_{AG} = \tau_{\text{ind}} \cdot \omega_s$$

$$- P_{RCL} = s \cdot P_{AG} \text{ (rotor copper)}$$

$$= P_{\text{conv}} = (1-s)P_{AG} = \tau_{\text{ind}} \cdot \omega_m$$

$$- P_{F\&W} \text{ (friction/windage)}$$

$$= P_{\text{out}} = \tau_{\text{load}} \cdot \omega_m$$

Sync Motor Phasor — Step by Step

1. $V_\phi = V_T / \sqrt{3}$ (Y) or V_T (Δ); set as $\angle 0^\circ$ reference
2. $P_{in} = P_{mech}(\text{hp} \times 746) + \text{all losses}$
3. $I_A = P_{in} / (3V_\phi \cdot \text{PF})$; lagging $\rightarrow \angle -\theta$, leading $\rightarrow \angle +\theta$
4. $E_A = V_\phi - I_A \cdot jX_S$ (complex arithmetic)
5. Load \uparrow : $|E_A|$ constant; $P_2/P_1 = \sin\delta_2/\sin\delta_1 \rightarrow \text{new } \delta \rightarrow \text{new } I_A$
6. Field \uparrow : P constant ($E_A \sin\delta = \text{const}$); scale $|E_A| \rightarrow \text{new } \delta \rightarrow \text{new } I_A$
7. Leading PF if I_A has positive angle vs V_ϕ